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Numerical analysis of sample dimensions in hot wire thermal conductivity measurements

Joaquim de Sylos Cintra Jr. *, Wilson Nunes dos Santos

Materials Engineering Department, Federal University of São Carlos CP 676, CEP 13565-905 São Carlos - SP, Brazil

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Abstract

The parallel hot wire technique has been widely used for experimental measurements of thermal conductivity of ceramic materials. The theoretical model that is used in the fitting procedure assumes that the sample behaves as an infinite medium. The finite dimensions of the actual samples are a source of potential errors. In this work temperature transients in finite samples are numerically simulated and the ones at the measuring point (Mp) are used to calculate thermal conductivities which are compared to the exact values. Consequently, the errors involved, due to the sample finite dimensions can be estimated. Since moving the measuring point further away from the external boundaries of the sample diminishes those errors, the positioning of the temperature sensor is numerically investigated. It is shown that for (r/w)(r = distance from wire to Mp; w = sample width) in the range of 0.2–0.3, the differences in the thermal conductivity values are less than 10%. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Simulation; Test methods; Thermal conductivity; Thermal properties

1. Introduction

In the past three decades the hot wire technique has been widely used for the experimental determination of the thermal conductivity of ceramic materials.[1–5] This technique is based upon the experimental detection, with a small thermocouple, of the thermal transient in a material specimen that appears due to the sudden electric heating of a wire placed inside the sample.[6–8] The usual experimental arrangement of that technique is schematically shown in Fig. 1. The test sample is composed of two rectangular blocks of the ceramic material that are joined together in such a way as to hold both the heating wire and the thermocouple joint (Mp = measuring point in Fig. 1). The experimental determination of the thermal conductivity of the tested material is done starting from the analysis of the temperature transient registered by the thermocouple. However, the basic theory used in these calculations assumes that from the point of view of the temperature sensor, the material in the sample can be considered as a infinite medium.[9] This assumption implies that the temperature transient that is picked up by the thermocouple joint, at the Mp, during the experiment cannot be altered by the fact that the actual sample has finite dimensions. In short, the heat lost from the external surfaces of the test specimen cannot significantly alter the temperature rise at the (Mp in Fig. 1). These considerations mean some restrictions in the applicability of the hot wire technique in terms of possible sample sizes and thermal conductivity allowable ranges.

Therefore the main purpose of this work is to investigate how and to what extent the finite dimensions of the actual test specimen can affect the results of the measurements of the material thermal conductivity using the thermal transient registered at the Mp.

In order to accomplish the task of evaluating the effects of dimensions of samples upon the thermal conductivity values that are obtained from the parallel hot wire technique, a numerical analysis of the temperature field within the sample is done, assuming finite dimensions and the heat losses through the external surfaces of the test specimen. With this procedure the temperature transient at the Mp is then obtained from a computer simulation. This numerically generated transient is then regarded as the one that would be obtained in a hypothetical hot wire experiment. Henceforth, if the

* Corresponding author.

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same non-linear regression analysis procedure[4] that is used in the hot wire parallel technique is applied to the hypothetical transient, a value for the fitted material thermal conductivity (k_{fit}) is obtained and can be compared to the assumed value (k_{num}) that was set in the transient simulation. The closeness or the distance between these two values (k_{fit}–k_{num}) for the material thermal conductivity will be a measure of the significance of the heat losses out of the external surfaces of the sample.

2. Background and model formulation

In order to carry out the proposed numerical analysis one considers a transverse section of the sample (section AA' in Fig. 1) passing through the measuring point (Mp). The dimensions of this transverse section of the sample are *w* (sample width) and *b* (sample thickness); *r* is the distance between the thermocouple and the hot wire. The spatial discretization of the transverse plane (*x,y*) is then done with a mesh of triangular elements with nodes (*i,j,k*) as shown in Fig. 2.

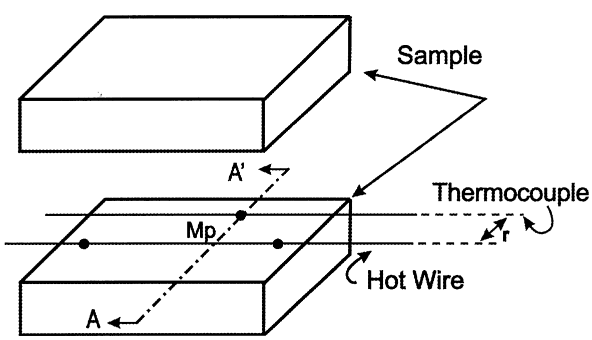

Fig. 1. Parallel hot wire technique. Experimental set-up.

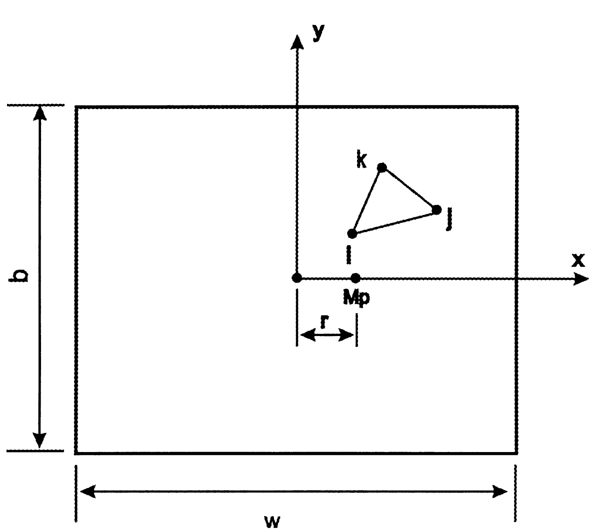

Fig. 2. Sample cross-section and system of coordinates for numerical calculations (Mp = measuring point).

The next step is to write down for each node a general balance equation that takes into account the contributions of heat transfer by conduction and by convection, internal heat generation and the time rate of change of the internal energy of the material associated with the node. Following this idea the resulting energy balance is represented by Eq. (1) for the (*i*) node.

$$\frac{\Delta T_{ki}}{R_{ki}} - \frac{\Delta \mathrm{T}_{ij}}{\mathrm{R}_{ij}} + \mathrm{q}''' S_i + \frac{1}{2} q''_{ij} L_{ij} + \frac{1}{2} q''_{ki} L_{ki} = \mathrm{CS}_i \frac{\Delta T_i}{\Delta t} \quad (1)$$

where:

ΔT_{ij} = temperature difference between points (*i*) and (*j*);
ΔT_{ki} = temperature difference between points (*k*) and (*i*);
R_{ij} = thermal resistance for heat conduction between (*i*) and (*j*);
R_{ki} = thermal resistance for heat conduction between (*k*) and (*i*);
q''' = rate of internal heat generation in triangular element (*i,j,k*);
S_i = area of element (*i,j,k*) associated to node (*i*);
q''_{ij} = heat flux at the surface defined by points (*i*) and (*j*);
q''_{ki} = heat flux at the surface defined by points (*k*) and (*i*);
L_{ij} = area per unit length of the surface defined by points (*i*) and (*j*);
L_{ki} = area per unit length of the surface defined by points (*k*) and (*i*);
C = heat capacity of the material within element (*i,j,k*);
$\Delta T_i/\Delta t$ = time rate of variation of temperature at node (*i*).

By applying the balance [Eq. (1)] to all nodes of the mesh, adopting a Crank–Nicholson[10] formulation for the time dependence, normalizing parameters and making the temperature dimensionless with the relation ($\theta = T/T_{ref}$; T_{ref} = reference temperature) one is able to obtain a set of algebraic Eq. (2). The solution of Eq. (2) is the seeked temperature field in the discretized sample.

$$[\mathrm{A}]\bar{\theta}^{\mathrm{k}+1} = [\mathrm{B}]\bar{\theta}^{\mathrm{k}} + \bar{\mathrm{p}} \quad (2)$$

where:

[A],[B] = coefficient matrices;
$\bar{\theta}^{k+1}$ = dimensionless temperature vector at time ($k+1$);
$\bar{\theta}^{k}$ = dimensionless temperature vector at time (k);
$\bar{p}$ = coefficient vector.

The theoretical model which is used in the parallel hot wire technique to measure thermal conductivities assumes that the temperature rise, at the measuring point (Mp), located at a distance (r) from the hot wire (linear heat source), is given by Eq. (3). In this equation it is implied the hypothesis of heat transfer throughout an infinite medium.[9] Therefore for the experimental analysis both the hot wire and the thermocouple are assumed to be embedded in a sample of infinite dimensions.

$$T(r,t) - \mathrm{T_o} = -\frac{q'}{4\pi k} Ei^{\left(-\frac{Cr^2}{4kt}\right)} \qquad (3)$$

where:

$T(r, t)$ = temperature at point (r) at time (t);
T_0 = initial temperature of the test sample;
q' = linear heat source at the wire;
r = distance from hot wire to MP;
t = time;
C = heat capacity of the material;
k = thermal conductivity of the material; and

$$-E_i(-x) = \text{exponential integral function} = \int_x^\infty \frac{\mathrm{e}^{-s}\mathrm{d}s}{s} \qquad (4)$$

In the non-linear regression procedure for the determination of the test material thermal conductivity, that is described in details by Santos and Cintra,[4] one minimizes the sum of the squares of the differences between the experimentally measured temperatures along the time transient at the measuring point and the values predicted by the theoretical model given by Eq. (3). The result of this fitting procedure is the thermal conductivity of the test specimen.

3. Numerical results and discussions

The system of Eq. (2) is then solved for a non-uniform mesh of 94 triangular elements with 65 nodes as shown in Fig. 3. As a reference case the values for w and b are set, respectively, to 100 and 60 mm because these are typical values that appear in parallel hot wire technique when applied to ceramic materials.

The heat source that generates the temperature transient within the material is computer simulated by assuming a prescribed value for the heat flux at the surfaces defined by the nodes that encircle the heating wire (nodes 1, 2, 3 and 4 in Fig. 3). The amount of electric current that is passed through the wire is usually adjusted in such a way as to create in the test specimen a temperature transient not so small that it can hardly be detected nor big enough to invalidate the basic assumption that the material properties are not affected by the temperature changes during the measurements. That means that in the experiments the material properties of the sample must be assumed as constant values. With that in mind, one has assumed for the heat flux at the heating surfaces (1–2, 2–3 and 3–4 in Fig. 3) the typical value of 3.5×10^4 W/m^2, for all numerical simulations.

The heat lost by convection at the external surfaces of the test sample can be calculated if one has the values for the convection heat transfer coefficients at those surfaces. In this work, since the medium in which the test specimens are placed is usually still air, we have assumed a value of 400 W/m^2 K for the convection heat transfer coefficient at all the external surfaces.

The initial temperature of the test material (T_o) was set to be equal to the ambient temperature (T_a) and the value 20°C was adopted for all numerical calculations.

The computer simulations were then carried out for a standard material with heat capacity of 2×10^6 J/m^{3}K and for assumed thermal conductivities varying from 0.15 W/mK up to 30 W/mK.

Fig. 4 shows the numerically calculated temperature transients at the node that coincides with the measuring point (Mp) in the hypothetical simulated heating experiment for different values of the material thermal conductivity. Actually in that figure the transient is shown as the normalized excess temperature at the measuring point node [Eq. (5)] versus time.

$$\Delta\theta_{\mathrm{MP}} = \frac{\mathrm{T_{MP}} - T_o}{T_a} \qquad (5)$$

where:

T_{Mp} = temperature at the Mp;
T_o = initial temperature of the sample;
T_a = ambient temperature.

It can be observed in Fig. 4 that at some time point the convection heat losses to the surroundings start affecting the assumption that the temperature at the Mp follows Eq. (3). These losses force the temperature at that point to go to a steady state value that surely is not predicted by the theoretical hot wire model given by Eq. (3). This effect can be clearly seen for the calculations done with the higher thermal conductivity values, for instance, k = 15 and 30 W/m K.

In the next step if one applies the non-linear regression procedure proposed by Santos and Cintra[4] in order to fit Eq. (3) to the linear part of the simulated temperature transient, numerical fitted values for the thermal conductivities can be obtained. In Table 1 (k_{fit}) are the fitted values that are obtained from the least squares fitting analysis and (k_{num}) are the values for the thermal conductivities that have been set in the computer simulations

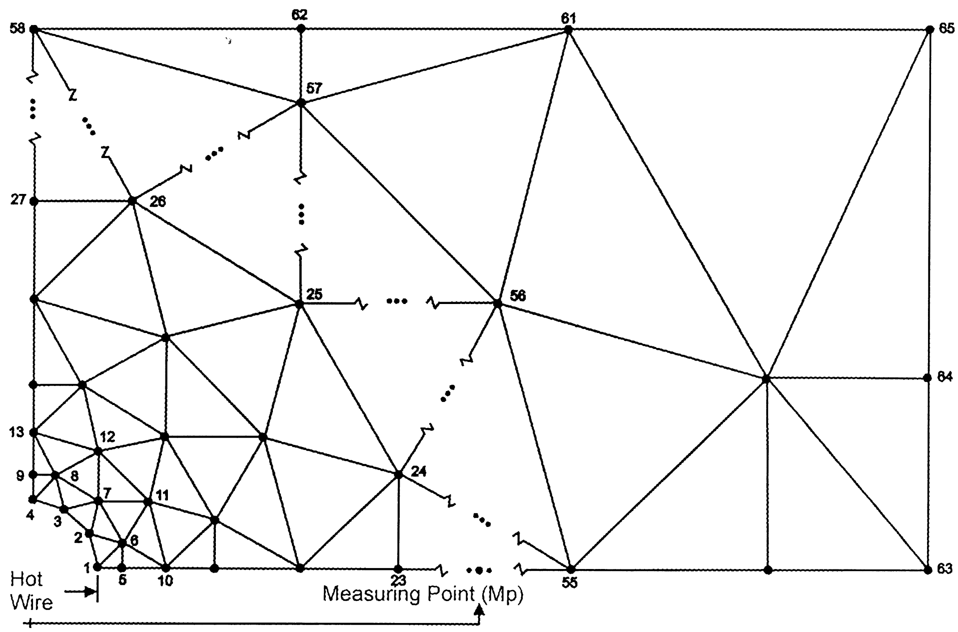

Fig. 3. Rectangular transverse section of sample with mesh of triangular elements for numerical calculations.

and which can, therefore, be regarded as the exact values. At this point it is important to note that in the theoretical model described by Eq. (3), if one considers a temperature difference given by $(T-T_o)/(q'/4\pi k)$ and a dimensionless time given by tk/CL_{ref}^2, where L_{ref} is some reference dimension, the function normalized temperature difference versus dimensionless time will always be the same whatever thermal conductivity is assumed for the test material. Then, in our simulated hot wire experiments, the differences in temperature profiles will be due to the effects of finite dimensions of the simulated test specimen. Therefore, as expected the % differences, i.e. the % errors in thermal conductivity values, will increase with the thermal conductivity as shown in Table 1.

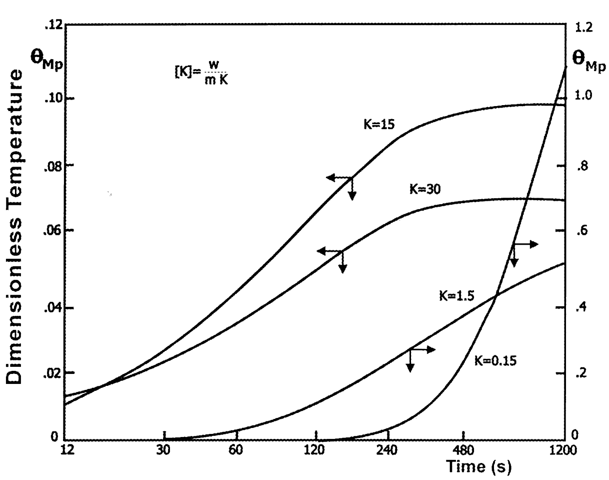

Fig. 4. Simulated temperature transients at the measuring point in parallel hot wire technique.

Now, it is also important to evaluate how these errors change with the position of the Mp in the finite sample. If one takes for the fitting procedure a temperature transient registered at a point further away from the external boundaries of the test sample it is expected that the difference between the fitted values for the thermal conductivities and the exact ones will decrease. That is exactly what is shown in Fig. 5 where the percent differences in thermal conductivity values are plotted against the distance from the Mp to the hot wire (r) made dimensionless with the width of the sample (w).

As we move the measuring point towards the external surface of the sample, we increase the ratio (r/w) and as a consequence the error in the measurement, by a fitting procedure to Eq. (3) of the material thermal conductivity will increase. The data points in Fig. 5 show exactly this behavior. As one can see in that figure, even for the high values of thermal conductivity, at least theoretically, it is possible to find a position for the measuring point in such a way as the fitting procedure will yield a reasonable agreement between the thermal

Table 1
Comparison between fitted and assumed values for thermal conductivities

Thermal conductivities		% Difference
Fitted values K_{fit}	Assumed values K_{num}	
0.142	0.150	5.3
1.407	1.500	6.2
11.878	15.0	20.8
20.821	30.0	30.6

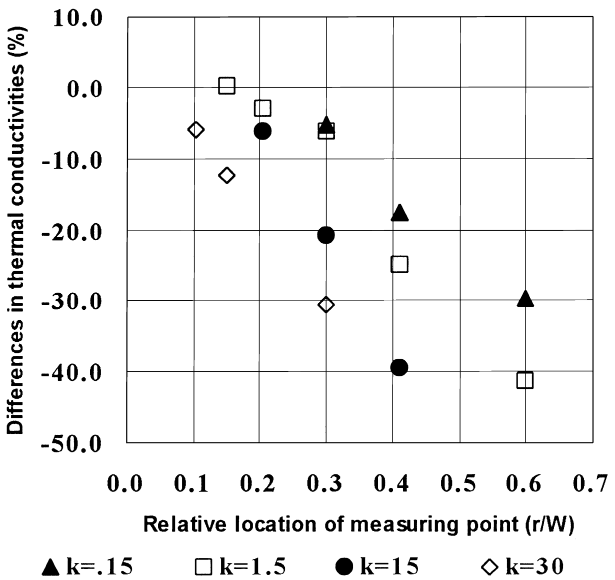

Fig. 5. Errors in thermal conductivity measurements versus the relative position of the measuring point (r/w).

conductivities exact and fitted. However, from the practical point of view, these measuring points set close to the hot wire for high thermal conductivity materials will be unsuitable in real experiments.

The results shown in Fig. 5 are extremely important to set the range of positions in which the measuring point can be placed within the sample if we know the order of magnitude of the thermal conductivity being measured with the parallel hot wire technique and the maximum error that can be accepted due to finite dimensions of the sample. For instance, one can see that for thermal conductivities up to around 1.5 W/m K, the typical used experimental positioning of the Mp which corresponds to (r/w) of approximately 0.2–0.3, will produce errors of less than 10%. Beyond this value, for the measuring distance to the hot wire, the influence and the effects of the finite dimensions of the sample will become more and more pronounced and will affect the thermal conductivity measurements with the parallel hot wire technique.

4. Conclusions

In this work temperature transients in a finite sample are numerically simulated and the one at the Mp is used to calculate thermal conductivities with the same fitting procedure that is used in the parallel hot wire technique.[4] The theoretical model that is used in the fitting procedure assumes that the sample behaves as an infinite medium. The finite dimensions of the actual samples are therefore a source of potential errors in those measurements, since the heat losses throughout external surfaces alter the temperature profiles within the sample. The fitted thermal conductivities are then compared to the exact ones and the errors involved which are due to the sample finite dimensions, are estimated. Also, since moving the location of the measuring point further away from the external boundaries of the test sample diminishes those errors, the positioning of the temperature sensor is also numerically investigated. It is shown that for (r/w) in the range of 0.2–0.3, the differences in the thermal conductivity values are less than 10% for test materials with thermal conductivities of up to around 1.5 W/m K.

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